

An Improved Formula for the Radiative Tail in Electron Scattering

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An exact formula for the radiative tail in electron scattering is presented in a form more convenient for numerical computation than previous versions.

One of the main problems in the analysis of deep inelastic electron scattering is the presence of large radiative tails, which constitute a substantial background and which must be subtracted from the observed cross section before the purely nuclear or hadronic cross section can be extracted.

Numerous calculations of these effects exist in the literature, the most complete of these being the reviews of Mo and Tsai¹, Tsai² and Maximon³. For further references see also the work of Berthot and Isabelle⁴ and ch. 7 of Ueberall's book⁵. Exact formulas for the radiative tail have been given by Maximon and Isabelle⁶ (neglecting target recoil) and by Mo and Tsai⁷. The latter have the exact result in terms of an integral over a photon angle in which the integral is rather sharply peaked at angles such that the photon is emitted parallel to either the incident or outgoing electron. It may be of some utility to experimentalists to separate the contribution of the peaks from the remainder of the radiative cross section in much the same way as was done by Maximon and Isabelle⁶ for potential scattering. We note here that neglect of the remainder amounts to the so-called peaking approximation. Once the separation is made, the dominant contribution to the radiative tail comes from the peaking terms. The background gives a significant (10–20%) correction but is given as an integral over a much smoother function than the full radiative cross section given in formula B.5 of Ref. 1, and it needs to be calculated somewhat less accurately than the full result. The separation is therefore convenient for numerical evaluation.

Our starting point will be formula B.5 of Reference 1. We begin here by reminding the reader of the notation used there and mentioning some useful kinematic identities.

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I. Notation

- $s(p)$ = four momentum of incident (outgoing) electron,
 p_i = four momentum of target,
 k = four momentum of emitted photon,
 p_f = four momentum of final hadronic or nuclear system,
 u = $p_i + k = s - p + p_i$,
 q = $s - p - k = p_f - p_i$.

M, M_f, m are masses of target, final nuclear state and electron, respectively

θ = scattering angle of electron,

$\theta_{k,s,p}$ = angle between u and k, s, p ,

we have

$$\begin{aligned} u_0 &= \varepsilon_s - \varepsilon_p + M, \\ u^2 &= M_i^2 + 2\omega(u_0 - |\mathbf{u}| \cos \theta_k), \\ q^2 &= 2m^2 - 2s \cdot p - 2\omega(\varepsilon_s - \varepsilon_p) + 2\mathbf{u} \cdot \mathbf{k}, \\ |\mathbf{u}|^2 &= (\mathbf{s} - \mathbf{p})^2, \quad \mathbf{u} \cdot \mathbf{s} = |\mathbf{s}|^2 - \mathbf{p} \cdot \mathbf{s}, \\ \mathbf{u} \cdot \mathbf{p} &= \mathbf{p} \cdot \mathbf{s} - |\mathbf{p}|^2, \end{aligned}$$

are independent of θ_k ; further $|\mathbf{u}| \sin \theta_p = |\mathbf{s}| \sin \theta$ and $|\mathbf{u}| \sin \theta_s = |\mathbf{p}| \sin \theta$. We then have, neglecting the electron mass in numerators of B.5

$$\begin{aligned} \nu &= \nu' \cong \{\omega[\varepsilon_s - \varepsilon_p - |\mathbf{u}| \cos \theta_k]\}^{-1}; \\ s \cdot p &\cong \varepsilon_s \varepsilon_p (1 - \cos \theta). \end{aligned}$$

Define $\omega_p = (u^2 - M_i^2)/2[M + \varepsilon_s(1 - \cos \theta)]$,
 $\omega_s = (u^2 - M_i^2)/2[M - \varepsilon_p(1 - \cos \theta)]$.

ω_s and ω_p are the maximum energies for photons emitted parallel to $\mathbf{s}$ (resp. $\mathbf{p}$).

II. Integration over $d(\cos \theta_k)$

Define

$$\begin{aligned} q_s^2 &= -2\varepsilon_p(\varepsilon_s - \omega_s)(1 - \cos \theta), \\ q_p^2 &= -2\varepsilon_s(\varepsilon_p + \omega_p)(1 - \cos \theta), \end{aligned}$$



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and note that for

$$\theta_k \cong \theta_p, \quad q^2 \cong q_p^2, \quad \nu \cong [\omega_p \varepsilon_s (1 - \cos \theta)]^{-1},$$

$$\omega \cong \omega_p, \\ a \cong -b \cong \omega_p \varepsilon_p \sin^2 \theta_p$$

while for

$$\theta_k \cong \theta_s, \quad q^2 \cong q_s^2, \quad \nu' \cong -[\omega_s \varepsilon_p (1 - \cos \theta)]^{-1},$$

$$\omega \cong \omega_s, \quad a' \cong -b' = \omega_s \varepsilon_s \sin^2 \theta_s$$

Next, in all terms involving $(a^2 - b^2)^{-1/2}$ add to and subtract from the numerators the corresponding quantities with $\theta_k \cong \theta_p$ and similarly for the terms in $(a'^2 - b'^2)^{-1/2}$ with $\theta_k \cong \theta_s$. The differences be-

tween exact and "peaked" integrand will be smooth while the first part can be integrated analytically, using the fact that

$$\begin{aligned} \int \frac{d(\cos \theta_k)}{(a^2 - b^2)^{1/2}} &= \frac{1}{\varepsilon_p \omega_p} \ln \frac{4 \varepsilon_p^2}{m^2}, \\ \int \frac{d(\cos \theta_k)}{(a'^2 - b'^2)^{1/2}} &= \frac{1}{\varepsilon_s \omega_s} \ln \frac{4 \varepsilon_s^2}{m^2}, \\ m^2 \int \frac{d(\cos \theta_k)}{(a^2 - b^2)^{3/2}} &= \frac{2}{\omega_p^3 \varepsilon_p \sin^2 \theta_p}. \end{aligned}$$

We obtain

$$\frac{d^2 \sigma}{d\Omega d\varepsilon_p} = P + B$$

where

$$\begin{aligned} P = \frac{\alpha^3}{\pi} \frac{M \varepsilon_p}{\varepsilon_s} \cos^2 \theta / 2 \left\{ \frac{\varepsilon_p \varepsilon_s}{\omega_s [M - \varepsilon_p (1 - \cos \theta)] q_s^4} \left(\frac{\varepsilon_s^2 + (\varepsilon_s - \omega_s)^2}{2 \varepsilon_s^2} \ln \frac{4 \varepsilon_s^2}{m^2} - \frac{\varepsilon_s - \omega_s}{\varepsilon_s} \right) \right. \\ \cdot \left[F(q_s^2) + \frac{2 G(q_s^2)}{M^2} \tan^2 \theta / 2 \right] + \frac{\varepsilon_p \varepsilon_s}{\omega_p [M + \varepsilon_s (1 - \cos \theta)] q_p^4} \\ \cdot \left(\frac{\varepsilon_p^2 + (\varepsilon_p + \omega_p)^2}{2 \varepsilon_p^2} \ln \frac{4 \varepsilon_p^2}{m^2} - \frac{\varepsilon_p + \omega_p}{\varepsilon_p} \right) \cdot \left[F(q_p^2) + \frac{2 G(q_p^2)}{M^2} \tan^2 \theta / 2 \right] \Big\} \end{aligned}$$

where F and G are defined as in the nonradiative cross section

$$\begin{aligned} \frac{d\sigma_{NR}(\varepsilon_s, \varepsilon_p, \theta)}{d\Omega d\varepsilon_p} &= \frac{\alpha^2 \varepsilon_p^2 \cos^2 \theta / 2}{q^4 \left[1 + \frac{\varepsilon_s}{m} (1 - \cos \theta) \right]} \left(F(q^2) + \frac{2 \tan^2 \theta / 2}{M^2} G(q^2) \right). \\ [q^2 &= -4 \varepsilon_s \varepsilon_p \sin^2 \theta / 2, \quad M_t^2 = M^2 + q^2 + 2 M (\varepsilon_s - \varepsilon_p)]. \end{aligned}$$

The quantity P corresponds to the peaking approximation and can be rewritten

$$P = \frac{\alpha}{\pi} \left\{ \frac{M + (\varepsilon_s - \omega_s) (1 - \cos \theta)}{\omega_s [M - \varepsilon_p (1 - \cos \theta)]} t_s \frac{d\sigma_{NR}(\varepsilon_s - \omega_s, \varepsilon_p, \theta)}{d\Omega d\varepsilon_p} + \frac{1}{\omega_p} t_p \frac{d\sigma_{NR}(\varepsilon_s, \varepsilon_p + \omega_p, \theta)}{d\Omega d\varepsilon_p} \right\}$$

with

$$t_s = \frac{1}{2} \left[1 + \left(\frac{\varepsilon_s - \omega_s}{\varepsilon_s} \right)^2 \right] \ln \frac{4 \varepsilon_s^2}{m^2} - \frac{\varepsilon_s - \omega_s}{\varepsilon_s}, \quad t_p = \frac{1}{2} \left[1 + \left(\frac{\varepsilon_p}{\varepsilon_p + \omega_p} \right)^2 \right] \ln \frac{4 \varepsilon_p^2}{m^2} - \frac{\varepsilon_p}{\varepsilon_p + \omega_p}.$$

Finally we have

$$\begin{aligned} B = \frac{\alpha^3}{2 \pi} \frac{M \varepsilon_p}{\varepsilon_s} \int_{-1}^1 d(\cos \theta_k) \left\{ \frac{\omega [2 G(q^2) / M^2 - F(q^2)]}{q^4 (u_0 - |\mathbf{u}| \cos \theta_k)} + (2 \omega)^{-1} [\varepsilon_p^2 (\cos \theta_p - \cos \theta_k)^2 \right. \\ + m^2 \sin^2 \theta_p \sin^2 \theta_k]^{-1/2} \left[\frac{\omega F(q^2)}{q^4 (u_0 - |\mathbf{u}| \cos \theta_k)} (-2 \nu s \cdot p [\varepsilon_s \varepsilon_p (1 + \cos \theta) + (\varepsilon_s - \varepsilon_p) \omega] \right. \\ + q^2 / 2 - s \cdot p + 2 (\varepsilon_s \varepsilon_p + \omega \varepsilon_s + \varepsilon_p^2)) - \frac{\omega_p F(q_p^2)}{q_p^4 [M + \varepsilon_s (1 - \cos \theta)]} \left(2 \varepsilon_p \left(\frac{\varepsilon_s \varepsilon_p}{\omega_p} (1 + \cos \theta) + \varepsilon_s - \varepsilon_p \right) \right. \\ \left. \left. + \frac{1}{2} q_p^2 / 2 - s \cdot p + 2 (\varepsilon_s \varepsilon_p + \omega_p \varepsilon_s + \varepsilon_p^2) \right) \right] + \frac{2 \omega G(q^2)}{M^2 q^4 (u_0 - |\mathbf{u}| \cos \theta_k)} [2 (s p)^2 \nu + s \cdot p - q^2 / 2] \\ \left. - \frac{2 \omega_p G(q_p^2)}{M^2 q_p^4 [M + \varepsilon_s (1 - \cos \theta)]} \left(\frac{2 \varepsilon_p}{\omega_p} s \cdot p + s \cdot p - q_p^2 / 2 \right) \right\} \end{aligned}$$

$$\begin{aligned}
& + (2\omega)^{-1} [\varepsilon_s^2 (\cos \theta_s - \cos \theta_k)^2 + m^2 \sin^2 \theta_s \sin^2 \theta_k]^{-1/2} \\
& \cdot \left[\frac{\omega F(q^2)}{q^4 (u_0 - |\mathbf{u}| \cos \theta_k)} (-2\nu s \cdot p [\varepsilon_s \varepsilon_p (1 + \cos \theta) + (\varepsilon_s - \varepsilon_p) \omega] + s \cdot p - q^2/2 - 2(\varepsilon_s \varepsilon_p - \omega \varepsilon_p + \varepsilon_s^2)) \right. \\
& - \frac{\omega_s F(q_s^2)}{q_s^4 [M - \varepsilon_p (1 - \cos \theta)]} \left(2\varepsilon_s \left(\frac{\varepsilon_s \varepsilon_p}{\omega_s} (1 + \cos \theta) + \varepsilon_s - \varepsilon_p \right) + s \cdot p - q_s^2/2 - 2(\varepsilon_s \varepsilon_p - \omega_s \varepsilon_p + \varepsilon_s^2) \right) \\
& + \frac{\omega G(q^2)}{M^2 q^4 (u_0 - |\mathbf{u}| \cos \theta_k)} [-2(s \cdot p)^2 \nu - s \cdot p + q^2/2] \\
& \left. - \frac{\omega_s G(q_s^2)}{M^2 q_s^4 [M - \varepsilon_p (1 - \cos \theta)]} \left(\frac{2\varepsilon_s}{\omega_s} s \cdot p - s \cdot p + q_s^2/2 \right) \right] \Bigg\}.
\end{aligned}$$

The radiative tail corresponding to elastic scattering plus bremsstrahlung gives the most important contribution to the total correction at large energy loss. Further this contribution is least reliably given by the peaking approximation. Thus, for $M_t = M$, and elastic form factors, one must use $P + B$, calculated for elastic scattering. The dominant contribution comes from $F(q_s^2)$ since we can have $q_s^2 \ll q^2$. If necessary the form factors in the background term can be given by a simple model, such as that for the Fermi distribution.

The contributions to the radiative tail for inelastic scattering accompanied by bremsstrahlung are generally smaller than for the elastic tail, except quite near the corresponding inelastic peak. It is therefore safer to use the peaking approximation for these contributions, although the background terms can be included here also if desired.

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